Important Questions

B.Tech IInd Sem, Maths – II (NAS -203)

Academic Session: 2013 -14

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1. (i) Find the complete solution of $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=xe^{3x}+\sin 2x$

(ii) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y+37\sin 3x=0$ find the value of y when x = $p/2$ being that y = 3, and $\frac{dy}{dx}=0$ when x = 0.

2. (i) Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$

(ii) Solve $(3x+2)^2\frac{d^2y}{dx^2}-(3x+2)\frac{dy}{dx}-12y=6x$

3. (i) Solve the following differential equation by reducing it into normal form

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$$

(ii) Solve by the change of independent variable $\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-\sin^2 x\, y=\cos x-\cos^3 x$

4. (i) Solve the simultaneous differential equation $\frac{dx}{dt}=-4x-4y,\frac{dx}{dt}+4\frac{dy}{dt}=-4y$ with x(0) = 1, y(0) = 0.

(ii) Apply the method of variation of parameter to solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$

5. (i) Find the Fourier Series expansion for f(x)= $x+\frac{x^2}{4},\ -p\le x\le p$

(ii) Find the Fourier Series expansion of $f(x)=\begin{cases}0, & -p<x\le 0\\ x, & 0<x<p\end{cases}$ hence find the sum of the series

$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.......$

6. (i) Find the Fourier series of the function $f(x)=\begin{cases}-x, & -p<x<0\\ x, & 0<x<p\end{cases}$

(ii) Find the Half Range Cosine series expansion of the function f(x) = x^2 in $0 < x < \pi$

7 (i) Find the Fourier series for the function $f(x)=\begin{cases}x, & 0<x<1\\ 1-x, & 1<x<2\end{cases}$ hence deduce that

$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{p^2}{8}$

(ii) Expand f(x) = x as Half range sine series in 0 < x < 2.

8(i) Solve the following P.D.E $x(y^2-z^2)p+y(z^2-x^2)q=z(x^2-y^2)$ where p, q have usual meaning.

(ii) Solve the P.D.E $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\cos mx\cos ny+30(2x+y)$

(iii) Solve the P.D.E $\left(D^2-D'^2-3D+3D'\right)z=xy+e^{x+2y}$

9 (i) Solve the P.D.E $(y^2+z^2)p-xyq=-zx$

(ii) Solve the P.D.E 4r – 4s + t = 16 log (x + 2y), where r, s, t have usual meaning.

(iii) Solve $(D-3D'-2)^2 z=2e^{2x}\tan(y+2x)$

10. (i) Classify the P.D.E $2\frac{\partial^2 u}{\partial t^2}+4\frac{\partial^2 u}{\partial x\partial t}+3\frac{\partial^2 u}{\partial x^2}=0$

(ii) Solve the following equation using the method of separation of variable $\frac{\partial^2 u}{\partial x\partial t}=e^{-t}\cos x$ given that u = 0 when t = 0 and $\frac{\partial u}{\partial t}=0$ when x = 0.

(iii) Solve the P.D.E by the separation of variable method , $u_{xx} = u_y + 2u$, u (0, y) = 0,

$\frac{\partial}{\partial x} u(0, y) = 1 + e^{-3y}$

11 (i) Find the deflection of the vibrating string of unit length whose end points are fixed. If the initial velocity is zero and the initial displacement is given by $u(x,0) = \begin{cases} 1, & 0 \le x \le 1/2 \\ -1, & 1/2 \le x \le 1 \end{cases}$.

(ii) Find the temperature distribution in a rod of length 2 m whose end points are fixed at zero temp and the initial temperature distribution id f(x) = 100x.

(iii) A thin rectangular plate lies in the xy plane defined by $0 \le x \le a, \quad 0 \le y \le b$. The edges x = 0 is kept at temperature f(y) while the remaining edge are kept at zero temperature. Find the steady state temperature inside the plate.

12. (i) Find the L.T of the function

(a) $f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$ (b) $f(t) = \begin{cases} e^{t-a}, & t > a \\ 0, & t < a \end{cases}$ (c) f(t) = $te^{-t}\sin 2t$ (d) $f(t) = \frac{e^{at} - \cos bt}{t}$

(e) Express the following function in terms of unit step function and hence find its L.T

$$f(t) = \begin{cases} 0, & 0 < t < 1 \\ t-1, & 1 < t < 2 \\ 1, & 2 < t \end{cases}$$

13. (i) Find the value of $\int_0^\infty e^{-3t} t \sin t dt$

(ii) Draw a graph and find the L.T of the following function of period 2a

$$f(t) = \begin{cases} \frac{h}{a} t, & 0 < t < a \\ \frac{h}{a}(2a - t), & a < t < 2a \end{cases}$$

(iii) Find the Inverse Laplace Transform of (a) $\frac{e^{-4s}(s+2)}{s^2 + 4s + 5}$ (b) $\cot^{-1}\left(\frac{s+3}{2}\right)$

(iv) Find the Inverse L.T of (a) $\frac{s}{(s+3)^2+4}$ (b) $\frac{3s+1}{(s-1)(s^2+1)}$

(v) Using the Convolution Theorem find the Inverse L.T of $\frac{1}{(s+2)^2(s-2)}$

14. (i) Using the L.T solve the following differential equation $\frac{d^2y}{dx^2}+n^2y=a\sin(nx+2)$

(ii) Apply L.T technique to solve $\frac{dx}{dt}+y=\sin t, \frac{dy}{dt}+x=\cos t$ x(0) = 2, y(0) = 0

15. (i) Solve in series the differential equation $2x^2y^{//}+xy^{/}-(x+1)y=0$ about x = 0

(ii) Solve in series the differential equation $xy^{//}+2y^{/}+xy=0$

16. (i) Express the polynomial $f(x)=4x^3-2x^2-3x+8$ in terms of Legendre function

(ii) P.T (i) $P_n^{/}(1)=\frac{1}{2}n(n+1)$ (ii) $P_n^{/}(-1)=(-1)^{n-1}\frac{n(n+1)}{2}$

(iii) P.T (a) $x^2J_n^{//}(x)=(n^2-n-x^2)J_n(x)+xJ_{n+1}(x)$

(b) $J_2^{/}(x)=\left(1-\frac{4}{x^2}\right)J_1(x)+\frac{2}{x}J_0(x)$ (c) $J_0+2J_1^2+2J_2^2+J_3^2+........=1$

(d) $J_{1/2}(x)=\sqrt{\frac{2}{px}}\sin x$