Department of Mathematics
Babu Banarasi Das National Institute of Technology & Management, Lucknow

RAS 301 Engineering Mathematics III: B. Tech. (Third Semester) – 2018-19
Assignment-4 (Unit-1) Complex Analysis

1. Write the necessary and sufficient condition for the analytic function.
2. Examine the nature of the function $f(z) = \frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}$, $z \neq 0$, $f(0) = 0$ in the region including the origin. [illegible]
3. What is harmonic function and hence show that $u(x, y) = \frac{1}{2}\log(x^2 + y^2)$ is harmonic. Find its harmonic conjugate.
4. If $w = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$. Determine the function ϕ.
5. If $u - v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ and $f(z) = u + iv$ is an analytic function of z, find f(z) in terms of z by Milne Thomson method.
6. If f (z) is regular function of z, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.
7. Evaluate using Cauchy's integral formula:
   (i) [illegible], if C is the circle $|z| = 3$
   (ii) Integrate $\frac{1}{(z^3 - 1)^2}$ the counter-clockwise sense around the circle $|z-1| = 1$.
   (iii) [illegible], if C is the circle $|z| = 3$.
8. Verify Cauchy theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points 1+i, -1+i and -1-i.
9. Expand $\frac{1}{(z+1)(z+3)}$ in Laurent series valid for (i) $1 < |z| < 3$ (ii) $|z| > 3$ (iii) $0 < |z+1| < 2$ and $|z| < 1$.
10. Expand the following function in a Laurent's series about $|z| = 0$, $f(z) = \frac{1 - e^{2z}}{z^4}$.
11. Discuss the singularity of $\frac{\cot \pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$ (ii) $f(z) = \frac{z - \sin z}{z^3}$ at $z = 0$.
12. Evaluate $\oint_C \frac{z-1}{(z+1)^2 (z-2)} dz$ where C is the circle $|z-i| = 2$ by the Cauchy residue theorem.
13. Evaluate $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$ where a>b.
14. Evaluate by contour integration method $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$.