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RAS 301 Engineering Mathematics III: B. Tech. (Third Semester) – 2018-19
Assignment-4 (Unit-1) Complex Analysis

1. Write the necessary and sufficient condition for the analytic function.

2. Examine the nature of the function $f(z) = \dfrac{xy^2(x+iy)}{x^2+y^4} : z \neq 0$, $f(0) = 0$ in the region including the origin.

3. What is harmonic function and hence show that $u(x,y) = \frac{1}{2}\log(x^2+y^2)$ is harmonic .Find its harmonic conjugate.

4. If $\omega = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \dfrac{x}{x^2+y^2}$. Determine the function ϕ .

5. If $u + v = \dfrac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ and $f(z) = u + iv$ is an analytic function of z, find f(z) in terms of z by Milne Thomson method.

6. If f (z) is regular function of z, show that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

7. Evaluate using Cauchy's integral formula

(i). $\int_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}\, dz$, if C is the circle $|z| = 3$

(ii) Integrate $\dfrac{1}{(z^3-1)^3}$ the counter clockwise sense around the circle |z-1|=1.

(iii) $\int_C \frac{e^z + \sin\pi z}{(z-1)(z-3)^2(z+4)}\, dz$, if C is the circle $|z| = 2$.

8. Verify Cauchy theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points 1+i, -1+i and -1-i.

9. Expand $\dfrac{1}{(z+1)(z+3)}$ in Laurent series valid for (i) $1 < |z| < 3$ (ii) $|z| > 3$ (iii) $0 < |z+1| < 2$ and $|z| < 1$.

10. Expand the following function in a Laurent's series about $|z| = 0$; $f(z) = \frac{1-\cos z}{z^3}$.

11. Discuss the singularity of $\dfrac{\cot \pi z}{(z-a)^2}$ at z = a and z = ∞ (ii) $f(z) = \dfrac{z - \sin z}{z^3}$ at z = 0.

12. Evaluate $\int_C \dfrac{z-1}{(z+1)^2(z-2)}\, dz$ where C is the circle |z-i|=2 by the Cauchy residue theorem.

13. Evaluate $\int_0^{\pi} \dfrac{d\theta}{a + b\cos\theta}$ where a>b.

14. Evaluate by contour integration method $\int_{-\infty}^{\infty} \dfrac{x^2}{(x^2+1)(x^2+4)}\, dx$.