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RAS 301 Engineering Mathematics III: B. Tech. (Third Semester) – 2018-19
Assignment-5 (Unit-5) Integral & z - Transform

1. Using Fourier integral representation, hence show that $e^{-|x|} = \frac{2}{\pi}\int_0^\infty \frac{\cos\lambda x}{\lambda^2+1}\,d\lambda$

2. Obtain Fourier cosine transform of $f(x) = \begin{cases} x, & for\ 0<x<1 \\ 2-x, & for\ 1<x<2 \\ 0, & for\ x>2 \end{cases}$

3. Find the Fourier sine transform of $F(x) = \begin{cases} 1-x^2, & if\ |x|<1 \\ 0, & if\ |x|>1 \end{cases}$ and use it to evaluate $\int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}\,dx$.

4. Using Fourier cosine integral representation of an appropriate function, show that $\int_0^\infty \frac{\cos\lambda x}{\lambda^2+1}\,d\lambda = \frac{\pi e^{-x}}{2}$.

5. Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$, for $x \ge 0, t \ge 0$ under the given conditions $u = u_0$ at $x = 0$, $t > 0$ with initial condition $u(x,0) = 0, x \ge 0$.

6. Use Fourier sine transform to solve the equation $\left(\frac{\partial u}{\partial t} = 2\left(\frac{\partial^2 u}{\partial x^2}\right)\right)$ under the conditions (i). u (0, t) = 0, (ii). $u(x, 0) = e^{-x}$, (iii) u(x, t) is bounded.

7. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the condition u(0, t) = 1, u(π, t) = 3, u(x, 0) = 1 for 0<x<π, t>0.

8. Find the z - transform of (a) $\frac{1}{z^2+3z+2}$ (b) $\frac{z^2+2z}{(z-1)^2(z-4)}$.

9. Find the inverse z-transform of $\frac{1}{(z-2)(z-3)}$ (a) |z|<2, (b) 2<|z|<3 and (c) |z|>3.

10. Evaluate $Z^{-1}\left[\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right]$

11. Use Z-transform to solve the difference equation $y_{k+2} - 2y_{k+1} + y_k = 3k+5$.

12. Using z-transform, solve the following difference equation $y_{k+2} + 4y_{k+1} + 3y_k = 3^k$, given that $y_0 = 0$ and $y_1 = 1$.

13. Solve using z-transform: $y_{k+2} - 2\cos\alpha\, y_{k+1} + y_k = 0$, $y_0 = 1, y_1 = 1$.