Department of Mathematics
Babu Banarasi Das National Institute of Technology & Management, Lucknow

RAS 301 Engineering Mathematics III: B. Tech. (Third Semester) – 2018-19
Assignment-5 (Unit-5) Integral & z - Transform

1. Using Fourier integral representation, $hence\ show\ that\ e^{-ax} = \frac{2a}{\pi}\int_0^\infty \frac{\cos\lambda x}{a^2+\lambda^2}$

2. Obtain Fourier cosine transform of $f(x) = \begin{cases} x, & for\ 0 < x < 1 \\ 2-x, & for\ 1 < x < 2 \\ 0, & for\ x < 2 \end{cases}$

3. Find the Fourier sine transform of
$F(x) = \begin{cases} 1-x^2, & if\ |x| < 1 \\ 0, & if\ |x| > 1 \end{cases}$ $.\ and\ use\ it\ to\ evaluate\ \int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\frac{x}{2}\,dx.$

4. Using Fourier cosine integral representation of an appropriate function, show that $\int_0^\infty \frac{\cos wx}{k^2+w^2}dw = \frac{-\pi e^{-kx}}{2k}$.

5. Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}\ for\ x \geq 0, t \geq 0$ under the given conditions $u = u_0\ at\ x = 0,$
$t > 0\ with\ initial\ condition\ u(x,0) = 0, x \geq 0$.

6. Use Fourier sine transform to solve the equation $\left(\frac{\partial u}{\partial x} = 2\left(\frac{\partial^2 u}{\partial x^2}\right)\right)$ under the conditions (i). u (0, t) = 0, (ii). U(x, 0) = e^{-x} , (iii) u(x, t) is bounded.

7. Solve $\frac{\partial v}{\partial x} = \frac{\partial^2 y}{\partial x^2}$ subject to the condition v(0, t) = 1, v(π, t) = 3, v(x, 0) = 1 for 0<x<π , t>0.

8. Find the z – transform of (a) $\frac{z}{z^2+7z+10}$ (b) $\frac{z^3-20z}{(z-2)^2(z-4)}$.

9. Find the inverse z–transform of $\frac{1}{(z-3)(z-2)}$ (a) $|z|$<2, (b) 2<$|z|$<3 and (c) $|z|$>3.

10. Evaluate $Z^{-1}\left[\frac{3z^2-18z+26}{(z-2)(z-3)(z-4)}\right]$.

11. Use Z-transform to solve the difference equation $y_{n+2} - 2y_{n+1} + y_n = 3n + 5$.

12. Using z-transform, solve the following difference equation
$y_{k+2} + 4y_{k+1} + 3y_k = 3^k\ ,\ given\ that\ y_0 = 0\ and\ y_1 = 1.$

13. Solve using z-transform: $y_{x+1} - 2\cos\alpha\, y_{x+1} + y_x = 0,\ y_0 = 1, y_1 = 1$.